

Student: _____
Date: _____

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Course: honors math

Assignment: FINAL EXAM MAT2270

1. Classify the graphs described in parts (a) and (b). Follow your decision with an explanation.



- (a) The graph shown on the right.
(b) A graph with six vertices, all of degree 4.

- (a) Determine whether the graph shown has an Euler circuit or path. Choose the correct answer below.

- ☐ A. The graph has an Euler path.
☐ B. The graph has an Euler circuit.
☐ C. The graph has both an Euler circuit and an Euler path.
☐ D. There is not enough information to classify the graph.
☐ E. The graph has neither an Euler circuit nor an Euler path.

Explain your reasoning. Choose the correct answer below.

- ☐ A. The graph has all even vertices.
☐ B. The graph has exactly one odd vertex.
☐ C. The graph has more odd vertices.
☐ D. The graph may or may not be connected.
☐ E. The graph has exactly two odd vertices

- (b) Determine whether a graph with six vertices, all of degree 4, has an Euler circuit or path. Choose the correct answer below.

- ☐ A. The graph has neither an Euler circuit nor an Euler path.
☐ B. The graph has an Euler circuit.
☐ C. The graph may or may not have an Euler path.
☐ D. The graph may or may not have an Euler circuit.
☐ E. The graph has an Euler path.

Explain your reasoning. Choose the best explanation below.

- ☐ A. The graph has exactly one odd vertex.
☐ B. All the vertices have odd degrees.
☐ C. The graph has more than two even vertices.
☐ D. The graph may or may not be connected.
☐ E. The graph has exactly two vertices with odd degrees.

2. The given table shows the preference schedule for an election with four candidates (A, B, C, and D). Use the Borda count method to solve parts a and b.

Number of Voters	7	5	4	2	2	2	1
1st	A	C	B	D	B	B	C
2nd	D	B	A	C	A	C	D
3rd	B	D	D	A	C	D	B
4th	C	A	C	B	D	A	A

(a) Find the winner of the election. Choose the correct answer below.

- ☐ Candidate A
☐ Candidate D
☐ Candidate B
☐ Candidate C

(b) Find the complete ranking of the candidates. Choose the correct answer below.

- ☐ A, D, C, B
☐ B, A, D, C
☐ B, D, A, C
☐ D, C, B, A

3. The given table shows the preference schedule for an election with four candidates (A, B, C, and D). Use the plurality-with-elimination method to solve parts a and b.

Number of Voters	29	16	12	8	6	1
1st	B	A	C	D	C	C
2nd	D	C	D	A	A	A
3rd	C	D	A	C	B	D
4th	A	B	B	B	D	B

(a) Find the winner of the election.

By the plurality-with-elimination method, the winner is candidate (1) _____

(b) Find the complete ranking of the candidates. Choose the correct answer below.

- ☐ C, D, A, B
☐ A, B, D, C
☐ A, B, C, D
☐ B, C, D, A

- (1) ☐ A.
☐ B.
☐ C.
☐ D.

4. The given table shows the preference schedule for an election with 5 candidates. Find the complete ranking using the method of pairwise comparisons. (Assuming that ties are broken using the results of the pairwise comparisons between the tying candidates.)

Number of Voters	14	11	9	6	5	4
1st	C	D	E	A	E	A
2nd	B	B	C	D	A	C
3rd	E	A	D	C	B	D
4th	D	E	A	B	D	E
5th	A	C	B	E	C	B

Choose the correct answer below.

- ☐ D, B, A, C, E
☐ D, E, A, C, B
☐ B, A, C, E, D
☐ E, C, D, B, A

5. Consider the weighted voting system $[q: 19, 13, 11, 9, 5]$.

- (a) What is the smallest value that the quota q can take?
 (b) What is the largest value that the quota q can take?
 (c) What is the value of the quota if *at least* two-thirds of the votes are required to pass a motion?
 (d) What is the value of the quota if *more* than two-thirds of the votes are required to pass a motion?

(a) The smallest value that the quota q can take is _____.

(b) The largest value that the quota q can take is _____.

(c) The quota if *at least* two-thirds of the votes are required is _____.

(d) The quota if *more* than two-thirds of the votes are required is _____.

6. Consider the weighted voting system [14: 12, 9, 4, 1].

- (a) Which players are critical in the coalition $\{P_2, P_3, P_4\}$?
 (b) Write down all winning coalitions.
 (c) Find the Banzhaf power distribution of this weighted voting system.

(a) Which players are critical in the coalition $\{P_2, P_3, P_4\}$?

- ☐ A. P_2, P_3 and P_4
☐ B. P_2 only
☐ C. P_3 and P_4 only
☐ D. P_2 and P_3 only

(b) Write down all winning coalitions. Choose the correct answer below.

- ☐ A. $\{P_1, P_2\}, \{P_1, P_3\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_4\}, \{P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4\}$
☐ B. $\{P_1, P_2\}, \{P_1, P_3\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_4\}, \{P_1, P_3, P_4\}, \{P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4\}$
☐ C. $\{P_1, P_2\}, \{P_1, P_4\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_4\}, \{P_1, P_3, P_4\}, \{P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4\}$
☐ D. $\{P_1, P_2\}, \{P_1, P_4\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_4\}, \{P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4\}$

(c) Find the Banzhaf power distribution of this weighted voting system.

$\beta_1 = \underline{\hspace{2cm}}, \beta_2 = \underline{\hspace{2cm}}, \beta_3 = \underline{\hspace{2cm}}, \beta_4 = \underline{\hspace{2cm}}$
 (Type integers or simplified fractions.)

7. Using set notation, write out the sample space for each of the following random experiments.

- (a) A coin is tossed four times in a row. The observation is how the coin lands (H or T) on each toss.
 - (b) A basketball player shoots three consecutive free throws. The observation is the result of each free throw (s for success, f for failure).
 - (c) A coin is tossed four times in a row. The observation is the number of times the coin comes up tails.
 - (d) A basketball player shoots four consecutive free throws. The observation is the number of successes.
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(a) Choose the correct answer below.

- ☐ A. $\{0, 1, 2, 3\}$
- ☐ B. $\{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT\}$
- ☐ C. $\{0, 1, 2, 3, 4\}$
- ☐ D. $\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

(b) Choose the correct answer below.

- ☐ A. $\{0, 1, 2, 3, 4\}$
- ☐ B. $\{0, 1, 2, 3\}$
- ☐ C. $\{ssss, sssf, ssfs, sfss, fsss, ssff, sfsf, fssf, sffs, fsfs, ffss, sfff, fsff, ffsf, fffs, ffff\}$
- ☐ D. $\{sss, ssf, sfs, fss, sff, fsf, ffs, fff\}$

(c) Choose the correct answer below.

- ☐ A. $\{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT\}$
- ☐ B. $\{0, 1, 2, 3, 4\}$
- ☐ C. $\{0, 1, 2, 3\}$
- ☐ D. $\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

(d) Choose the correct answer below.

- ☐ A. $\{ssss, sssf, ssfs, sfss, fsss, ssff, sfsf, fssf, sffs, fsfs, ffss, sfff, fsff, ffsf, fffs, ffff\}$
 - ☐ B. $\{0, 1, 2, 3, 4\}$
 - ☐ C. $\{0, 1, 2, 3\}$
 - ☐ D. $\{sss, ssf, sfs, fss, sff, fsf, ffs, fff\}$
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8. The 1270 students at a certain high school are having an election for homecoming king. The candidates are Tomlinson (captain of the football team), Garcia (class president), and Marsalis (member of the marching band). At the football game a week before the election, a pre-election poll was taken of students as they entered the stadium gates. Of the students who attended the game, 195 planned to vote for Tomlinson, 47 planned to vote for Garcia, and 133 planned to vote for Marsalis. Complete parts (a) through (d) below.

(a) Describe the sample for this survey. Choose the correct answer below.

- ☐ A. The sample is all of the students at the football game.
- ☐ B. The sample is all of the students who voted for Garcia.
- ☐ C. The sample is all of the students at the high school.
- ☐ D. The sample is all of the students who voted for Tomlinson.
- ☐ E. The sample is all of the students who voted for Marsalis.

(b) Give the sampling proportion for this survey.

The sampling proportion is _____ %.
(Round to the nearest integer as needed.)

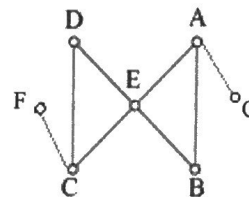
(c) Do you think the survey was subject to selection bias? Explain.

- ☐ A. No, because the survey was taken at a football game. Therefore, any student could vote for any candidate equally.
- ☐ B. No, because each student who attends the high school. Therefore, the sampling frame is the same as the target population.
- ☐ C. Yes, because the survey was taken at a football game. Therefore, students who did not attend the football game are not included in the sample.
- ☐ D. Yes, because the survey was taken at a football game. Therefore, students who attended the football game are included in the sample.

(d) Was the sampling method used in this survey the same as quota sampling? Explain.

- ☐ A. No, because the sample was not chosen based on meeting certain quotas (age, race, income level, etc).
- ☐ B. Yes, because the sampling frame and the target population are the same.
- ☐ C. No, because the sample was not representative of the high school population.
- ☐ D. Yes, because the sample was chosen based on meeting certain quotas (age, race, income level, etc).

9. Explain why the graph to the right has no Hamilton circuit but does have a Hamilton path.



Why does the graph have no Hamilton circuit?

- ☐ A. There is no Hamilton circuit since no matter where one starts, one has to cross F twice to visit every vertex and get back to the starting vertex.
- ☐ B. There is no Hamilton circuit since there are less than 10 vertices.
- ☐ C. There is no Hamilton circuit since the graph is not a complete graph, in which every vertex is connected to every other vertex.
- ☐ D. There is no Hamilton circuit since no matter where one starts, one has to cross E twice to visit every vertex and get back to the starting vertex.

Why does the graph have a Hamilton path?

- ☐ A. There is a Hamilton path since the graph is not a complete graph, in which every vertex is connected to every other vertex.
- ☐ B. There is a Hamilton path since there is a path starting at A that visits all the vertices of the graph once and only once.
- ☐ C. There is a Hamilton path since there is a path starting at F that visits all the vertices of the graph once and only once.
- ☐ D. There is a Hamilton path since there are less than 10 vertices.

10. (a) How many edges are there in K_{10} ?
- (b) How many edges are there in K_{12} ?
- (c) If the number of edges in K_{49} is x , and the number of edges in K_{50} is y , what is the value of $y - x$?

(a) The number of edges in K_{10} is _____.

(b) The number of edges in K_{12} is _____.

(c) The value of $y - x$ is _____.